**Colorado Technical University**

**Course:** MATH205 – Differential Calculus

#### Unit 6 Part 11 Readings – Extrema

**Extrema**

The **maximum** or **minimum** over the entire function is called an "Absolute" or

"**Global**" **maximum** or **minimum**. There is only one **global**

**maximum** (**and** one **global minimum**) but there can be more than one **local**

**maximum** or **minimum**.

**Maximums and minimums** occur where the slope = 0 or is undefined changing from

If *f*(*x*)' changes from positive to negative at point x = c, then the curve is

increasing to decreasing and c is a relative maximum

If *f*(*x*)' changes from negative to positive at point x = c, then the curve is changing from

decreasing to increasing and c is a relative minimum

If *f*(*x*)' does not change sign at point x = c, then c is neither a relative maximum nor

minimum

**Inflection Points**

An **inflection point** is a **point** on the graph of a function at which the concavity changes

**Points** of **inflection** can occur where the second derivative is zero

Solve f '' = 0 to find the potential **inflection points**.

**Concavity**

Geometrically, a function is concave upward on an interval if its graph behaves like a

portion of a parabola that opens upward. Likewise, a function that is concave

downward on an interval looks like a portion of a parabola that opens downward. If the graph of a function is linear on some interval in its domain, its second derivative

will be zero, and it is said to have no concavity on that interval.

A function is said to be **concave** upward on an **interval** if f″(x) > 0 at each point in

the **interval** and **concave** downward on an **interval** if f″(x) < 0 at each point in

the **interval**

In determining intervals where a function is concave upward or concave downward, first

find domain values where f″(x) = 0 or f″(x) does not exist. Then test all intervals around these values in the second derivative of the function. If f″(x) changes sign, then ( x, f(x)) is a point of inflection of the function.

As with the First Derivative Test for Local Extrema, there is no guarantee that the

second derivative will change signs, and therefore, it is essential to test each interval around the values for which f″(x) = 0 or does not exist.

**Monotonicity**

 A function is called “**increasing**” (or **non-decreasing**) if its values are only rising and

never falling with increasing values of x

It is **strictly increasing** if values always become larger and cannot be constant

A function is called “**decreasing**” (or **non-increasing**) if its values are only falling and

never rising with increasing values of x

It is **strictly decreasing** if values always become smaller and cannot be constant

If a function f(x) is differentiable on the **interval** (a,b) and belongs to one of the four

considered types (i.e. it is increasing, strictly increasing, decreasing, or strictly

decreasing), this function is called **monotonic on this interval**

To determine monotonicity, we take the root of the derivative (where dy/dx=0)

If the derivative has at least one root, the entire function cannot be strictly increasing or

strictly decreasing, but we can still determine its monotonicity in the intervals

between the roots by evaluating the derivative equation at a point in the interval

 If there is no root, evaluate the derivative equation at any point to determine that the

function is strictly increasing if it is positive, or strictly decreasing if it is negative

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