**Colorado Technical University**

**Course:** MATH207 – Integral

**Unit 3 Part 6 Readings: Volumes**

**Area** A **under a curve** *y* = *f*(*x*) from *x* = *a* to *x* = *b* is

lim

*n*→∞

A = sum of the *n* rectangles between *a* and *b*

= *∫ab* *f*(*x*) *dx*

*b*

*a*

= F(*x*) |

= F(*b*)  F(*a*)

where F(*x*) = *∫* *f*(*x*) *dx* note: no constant of integration!

### Definite Integral

If *f*(*x*) is a continuous function between *x* = a and *x* = b and the

derivative of F(*x*) = F'(*x*) is *f*(*x*) then

*∫ab* *f*(*x*) *dx* = F(*b*) - F(*a*)

*b*

*a*

= F(*x*) |

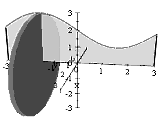
*Factoid: ∫ab* *f*(*x*) *dx* = *∫ba* *f*(*x*) *dx*

**Area between two curves** *f* **and** *g*

A = *∫ab* *f*(*x*) - *g*(*x*) *dx*

**Volumes of Revolution**

Find the volume of a solid created by revolving a function about an axis

**Disk method**

infinitesimally thin circular slices added together to

get the volume

the formula for calculating the volume of a solid

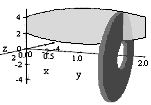
formed by revolving a function *f*(*x*) about the

*x*-axis axis is:

***π****∫ab*[ *f*(*x*)] 2 *dx*

the formula for calculating the volume of a solid

formed by revolving a function *f*(*y*) about the *y*-axis axis is:

***π****∫ab*[ *f*(*y*)] 2 *dy*

# Ring Method

### When two curves are rotated about an axis, the

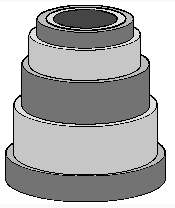
### resulting shape can be sliced into an

### infinitesimally-thin set of ring-shaped

### (washer, donut, lifesaver, x-mas wreath) slices

**Shell Method**

Uses infinitesimally-thin cylindrical shells

instead of disks or washers

*V* = sum of concentric cylindrical shells

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | radius | height | thickness | shells parallel to |
| *V* = | 2 ***π*** *∫ab x* | *f*(*x*) | *dx* | *y*-axis |
| *V* = | 2 ***π*** *∫ab y* | *f*(*y*) | *dy* | *x*-axis |

