**Colorado Technical University**

 **Course:** MATH207 – Integral Calculus

#### Unit 5 Part 9 Readings: Partial Derivatives

# Partial Derivatives

If z = ƒ (x,y) then partial derivatives will be used

This function must be such that for every pair of x- and y- values

there exists a unique z



Common notations: ∂z/∂x ∂z/∂y ƒx ƒx (x,y)

When **evaluating a function** at ∂z/∂x

y is treated as a constant (held steady)

only x is allowed to vary

(8y = 8yx0 ) with a derivative of 0 \* 8 \* x \* y = 0

Consider z as a function with only one variable, x, and find

the derivative as normal with y considered a constant

For ∂z/∂y hold *x* steady and vary *y*

Full derivative - combine and multiple by appropriate partial

Once the partial derivative is found, evaluate each partial by

replacing the variable with its value from (x, y, z) and solve

Each partial derivative will return the slope of a line tangent to the

surface at that three-space point parallel to the respective

axis

**Higher-Order Partial Derivatives**

Remember, we had higher-order derivatives with functions of one variable?

They looked like:

 y" = $\frac{d^{2}y}{dx^{2}}$

You could keep going: y’’’ y’’’’ …

We will also have higher-order derivatives of functions of more than one variable

In single variable calculus we saw that the second derivative is often useful: in

appropriate circumstances it measures acceleration; it can be used to identify

maximum and minimum points; it tells us something about how sharply curved a graph is

Not surprisingly, second derivatives are also useful in the multi-variable case, but again

not surprisingly, things are a bit more complicated

It’s easy to see where some complication is going to come from: with two variables

there are four possible second derivatives

To take a “derivative,” we must take a partial derivative with respect to x or y, and there

are four ways to do it: x then x, x then y, y then x, y then y

For a function of two variables ƒ(x,y), there will be a total of four possible second-order

derivatives:

ƒxx = $\frac{∂^{2}ƒ}{∂x^{2}}$

ƒyy = $\frac{∂^{2}ƒ}{∂y^{2}}$

ƒxy = $\frac{∂^{2}ƒ}{∂y∂x}$

ƒyx = $\frac{∂^{2}ƒ}{∂x∂y}$

The last two second-order partial derivatives are often called **mixed partial derivatives**

because we are taking derivatives with respect to more than one variable

Note that the-order that we take the derivatives in is given by the notation:

 ƒxy means first differentiate with respect to x then with respect to y

 $\frac{∂^{2}ƒ}{∂y∂x}$ means first differentiate with respect to x then with respect to y

Example: Find all the second-order derivatives for exy

Solution

First find the first-order partial derivatives:

ƒx = $\frac{∂(e^{xy})}{∂x}$ = yexy

ƒy = $\frac{∂(e^{xy})}{∂y}$ = xexy

Now, let’s get the second-order derivatives:

ƒxx = $\frac{∂^{2}(e^{xy})}{∂x^{2}}$ = $\frac{∂(ye^{xy})}{∂x}$ = y2exy

ƒyy = $\frac{∂^{2}(e^{xy})}{∂y^{2}}$ = $\frac{∂(xe^{xy})}{∂y}$ = x2exy

ƒxy = $\frac{∂^{2}(e^{xy})}{∂y∂x}$ = $\frac{∂(ye^{xy})}{∂y}$ = xyexy + exy or exy(xy +1)

ƒyx = $\frac{∂^{2}(e^{xy})}{∂x∂y}$ = $\frac{∂(xe^{xy})}{∂x}$ = xyexy + exy or exy(xy +1)

Notice that ƒxy = ƒyx

This will be true for almost all of the functions you will run into in your life (discontinuous

functions are the exception to the rule)

If partials have the same value, they are not “distinct”

**Clairaut’s Theorem:** If the mixed partial derivatives are continuous, they are equal.

There are, of course, even higher-order partial derivatives as well

How many third-order partial derivatives would there be for a two-variable function?

ƒxxx ƒyxx ƒxyx ƒxxy ƒxyy ƒyxy ƒyyx ƒyyy

These methods will work for any number of variables as well – these will be VERY

complicated to solve