**Colorado Technical University**

**Course:** MATH207 – Integral Calculus

#### Unit 8 Part 15 Readings: Taylor and Fourier Series

#### Taylor Series

#### When x is large, many terms are needed to give an accurate approximation for *f* (x)

#### When x is less than 1, only a few terms are needed

#### A person with curly hair Description automatically generated with low confidenceThe Taylor series was invented to solve the problem for large

#### values of x

Start with an power series offset by "b":

#### *f* (x) = a0 +a1(xb) + a2(xb)2 + a3(xb)3 + …

#### Pick a value for b that is very close to the value of x (usually the

#### same value)

#### The complete Taylor series is:

*f* (x) =*f* (b) + *f* '(b) x + *f* ' '(b) x2/2 + *f* ' ' '(b) x 3/3! + *f* ' ' ' '(b) x 4/4! …

#### Rule: If a series is needed for a function whose x values will

#### always be less than one, use the Maclaurin series. For

#### functions whose x values are larger than one, use the

#### Taylor series with a value for "b" near the values of x you

#### will be using.

Computation of Taylor and Maclaurin series coefficients can be Brook Taylor

messy because of the need to find several derivatives of

derivatives of derivatives.

The easiest way to develop a series for functions which are more complicated

than the basic elementary functions is to use the operations of addition,

subtraction, multiplication, division, differentiation and integration on the existing

series for the elementary functions.

TI89: F3 9 expr var #terms “b”

A picture containing text, person

Description automatically generated**Fourier Series**

One of the difficulties with Taylor series expansions is that, in general,

they can be used to represent a given function only for values

of *x* close to *a* [when expanded in powers of *(x* - *a)].*

A Fourier series expansion is often used when it is necessary to

approximate a function over a larger interval of values of *x.*

Fourier series are also specifically designed to estimate non-continuous waves

Fourier discovered any periodic function could be written as an

infinite series of sine and cosine terms

As you add sine waves of increasingly higher frequency, the approximation gets better

“Even” functions are represented by cosines (which are even)

“Odd” functions are represented by sines (which are odd)

“Arbitrary” functions include both

Fourier series can be expressed either as “trigonometric” or “exponential”

**Trigonometric:**

∞

x(t) = a0 + ∑ (ancos(nω0t)+bnsin(nω0t))

n=0

Chart

Description automatically generated**Exponential:**

∞

x(t) = a0 + ∑ (cnejnω0t)

n=0

a0 is a constant term

It is the average value of the

original function

This is often called the DC or the

zero frequency component of the Fourier series

The second component a1cos(ω0t) has

exactly one oscillation of the cosine in the period T=1

We call this the 1st or

fundamental harmonic

Adding a0 adjusts its vertical

location

The 2nd harmonic (n=2) has exactly

two oscillations in one period T=1 of the original function

Added to the zero-eth and first

harmonic still looks like a

cosine

The 3rd harmonic (n=3) has exactly

three oscillations in one

period, T=1, of the original function

Added to the earlier harmonics you begin to get some “fitting”

The 4th harmonic (n=4) has exactly four oscillations in one period, T=1, of the original function

Added to the earlier harmonics gives a better “fit”

There are certain limitations inherent in the use of the Fourier Series, but these are almost

never a problem in engineering applications

The Fourier series converges:

if ∫xT(t) dt < ∞ (as long as the function is not infinite over a finite interval)

if xT has a finite number of discontinuities in one period

if xT has a finite number of maxima and minima in one period

except at discontinuities, where it converges to the midpoint of the discontinuity

At a discontinuity there is an overshoot (Gibb's phenomenon - about 9% for a square

wave). However this discontinuity becomes vanishingly narrow (and it's area, and

energy, are zero), and therefore irrelevant as we sum up more terms of the

series

Four of the basic periodic waves that commonly occur in the analysis of electric and mechanical systems are shown

Diagram

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**Summary of popular forms of the Fourier transform**

**ordinary frequency ξ (hertz) – unitary**



**angular frequency ω (rad/s) – non-unitary**



**angular frequency ω (rad/s) – unitary**

\displaystyle \hat{f}_3(\omega) \ \stackrel{\mathrm{def}}{=}\  \frac{1}{(2 \pi)^{n/2}} \int_{\mathbb{R}^n} f(x) \ e^{-i \omega\cdot x}\, dx = \frac{1}{(2 \pi)^{n/2}} \hat{f}_1\left(\frac{\omega}{2 \pi} \right) = \frac{1}{(2 \pi)^{n/2}} \hat{f}_2(\omega) 



Diagram

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