**Colorado Technical University**

**Course:** MATH207 – Integral Calculus

#### Unit 9 Part 17 Readings: Polars

**Parametric equations**

Equations that specify *x* and *y* in terms of a third variable *t* called a parameter (often

representing time)

Ex: the parametric equations: x = x0 + a cos bt and y = y0 + a sin bt

describe the circle (x x0)2 + (y –y0)2 = a2

Derivative for parameter curves: if x = g(t) and y = h(t)

dy/dx = dy/dt / dx/dt = h'(t)/g'(t)

Slopes of tangent lines dy/dx

# Polar graphs

ordered pair *P* = (*r*, *θ*)

*r* is the distance (radius) from the pole

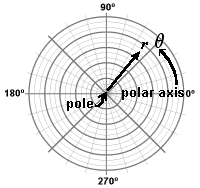
*θ* is an angle from the polar axis (degreesº or radiansπ)

Positive angles measured counterclockwise from polar axis

Negative angles measured clockwise from polar axis

*r* > 0 - point lies on the terminal side of *θ*

*r* < 0 - point lies along the ray opposite the terminal side of *θ*

*****r* = 0 point lies on the pole irrespective of the value of *θ*

# Polar to rectangular conversion

*x* = *r* cos *θ* *y* = *r* sin *θ*

## Rectangular to polar conversion

*r* =  *θ* = arctan (*y* ÷ *x*)

# Polar equations

use variables *r* and *θ*

**Converting rectangular equations to polar equations**

replace *x* with *r* cos *θ* and *y* with *r* sin *θ*

**Converting polar equations to rectangular equations**

not easy; try: *r* 2 = *x*2 + *y*2

*r* cos *θ* = *x*

*r* sin *θ* = *y*

Hint

First calculate

=

**Use the exact table above**

tan *θ* = *y*/*x*

Slope of a tangent line:

Let ƒ be a differentiable function at θ0

The slope of the line tangent to the curve

r = ƒ(θ) at the point (ƒ(θ0), θ0) is:

dy/dx =

provided the denominator is not zero at this point

At angles for which ƒ(θ0) = 0 and ƒ ' (θ0) ≠ 0,

the tangent line is θ = θ0 with slope tanθ0

### Area of Regions Bounded by Polar Coordinates

Let R be the region bounded by the graphs of r = ƒ(θ) and r = *g*(θ) between θ = *α* and θ = *β* where ƒ and *g* are continuous and ƒ(θ) ≥ *g*(θ) ≥ 0 on [*α*, *β*]

The area of R is: *∫αβ* .5(ƒ(θ)2 – *g*(θ)2) dθ

**Exact Values of Sin, Cos and Tan Functions**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Deg** | **Rad** | **sin** | **cos** | **tan** |
| 0 | 0 | 0 | 1 | 0 |
| 30 | *π*/6 | 1/2 | /2 | /3 |
| 45 | *π*/4 | /2 | /2 | 1 |
| 60 | *π*/3 | /2 | 1/2 |  |
| 90 | *π*/2 | 1 | 0 | undef |
| 120 | 2*π*/3 | /2 | –1/2 | – |
| 135 | 3*π*/4 | /2 | –/2 | –1 |
| 150 | 5*π*/6 | 1/2 | –/2 | –/3 |
| 180 | *π* | 0 | 1 | 0 |
| 210 | 7*π*/6 | –1/2 | –/2 | /3 |
| 225 | 5*π*/4 | –**/**2 | –/2 |  |
| 240 | 4*π*/3 | –/2 | –1/2 |  |
| 270 | 3*π*/2 | –1 | 0 | undef |
| 300 | 5*π*/3 | –/2 | 1/2 | – |
| 315 | 7*π*/4 | –/2 | /2 | –1 |
| 330 | 11*π*/6 | –1/2 | /2 | –/3 |
| 360 | 2*π* | 0 | 1 | 0 |

